

Impact of Neutrino Oscillation Measurements on Theory

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Abstract. Neutrino oscillation data had been a big surprise to theorists, and indeed they have ongoing impact on theory. I review what the impact has been, and what measurements will have critical impact on theory in the future.

INTRODUCTION

I was asked to comment on the impact of neutrino oscillation measurements on theory. It is completely clear that recent neutrino oscillation data had big impact on theory, and it will continue to do so. I will remind you about the ongoing impact. Then I will list measurements that will have critical impact on theory in the future.

Let me organize my discussion as “Past,” “Present,” and “Future.”

PAST

It is useful to recall why theorists had always been interested in the small neutrino masses and their consequences on neutrino oscillation. It is because we are always interested in probing physics at as high energies as possible. One way to probe it is of course to go to the high-energy collider experiments and study physics at the energy scale directly. Another way is to look for rare and/or tiny effects coming from the high-energy physics. The neutrino mass belongs to the second category.

To study rare and/or tiny effects from physics at high energies, we can always parameterize them in terms of the power series expansion,

$$\mathcal{L} = \mathcal{L}_4 + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \cdots. \quad (1)$$

The zeroth order term \mathcal{L}_4 is renormalizable and describes the Standard Model. On the other hand, the higher order terms are suppressed by the energy scale of new physics Λ . Possible operators can be classified systematically, which I believe was done first by Weinberg (but I couldn't find the appropriate reference). With two powers of suppression, there are many terms one can study:

$$\mathcal{L}_6 \supset QQQL, \bar{L}\sigma^{\mu\nu}W_{\mu\nu}He, W_\nu^\mu W_\lambda^\nu B_\mu^\lambda, \bar{s}d\bar{s}d, (H^\dagger D_\mu H)(H^\dagger D^\mu H), \cdots \quad (2)$$

The examples here contribute to proton decay, $g - 2$, anomalous triple gauge boson vertex, $K^0 - \bar{K}^0$ mixing, and the ρ -parameter, respectively. It is interesting that there is only one operator suppressed by a single power:

$$\mathcal{L}_5 = (LH)(LH). \quad (3)$$

After substituting the expectation value of the Higgs, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{\Lambda}(LH)(LH) \rightarrow \frac{1}{\Lambda}(L\langle H \rangle)(L\langle H \rangle) = m_\nu \nu \nu, \quad (4)$$

nothing but the neutrino mass.

Therefore the neutrino mass plays a very unique role. It is the lowest-order effect of physics at short distances. This is a very tiny effect. Any kinematical effects of the neutrino mass are suppressed by $(m_\nu/E_\nu)^2$, and for $m_\nu \sim 1$ eV which we now know is already too large and $E_\nu \sim 1$ GeV for typical accelerator-based neutrino experiments, it is as small as $(m_\nu/E_\nu)^2 \sim 10^{-18}$. At the first sight, there is no hope to probe such a small number. However, any physicist knows that interferometry is a sensitive method to probe extremely tiny effects. For interferometry to work, we need a coherent source. Fortunately there are many coherent sources of neutrinos in Nature, the Sun, cosmic rays, reactors (not quite Nature), etc. We also need interference for an interferometer to work. Because we can't build half-mirrors for neutrinos, this could have been a show stopper. Fortunately, there are large mixing angles that make the interference possible. We also need long baselines to enhance the tiny effects. Again fortunately there are many long baselines available, such as the size of the Sun, the size of the Earth, etc. Nature was very kind to provide all necessary conditions for interferometry to us! Neutrino interferometry, a.k.a. neutrino oscillation, is therefore a unique tool to study physics at very high energy scales.

Indeed, the recently established neutrino oscillation results [1, 2]

$$\Delta m_{atm}^2 \sim 0.002 \text{eV}^2, \quad (5)$$

$$\Delta m_{solar}^2 \sim 0.00007 \text{eV}^2, \quad (6)$$

interpreted naively in a ‘‘hierarchical’’ mass scheme

$$m_3 \sim \sqrt{\Delta m_{atm}^2} \sim 0.04 \text{eV}, \quad (7)$$

$$m_2 \sim \sqrt{\Delta m_{solar}^2} \sim 0.008 \text{eV}, \quad (8)$$

suggests

$$\Lambda \sim \frac{\langle H \rangle^2}{m_3} \sim 8 \times 10^{14} \text{ GeV}. \quad (9)$$

It is tantalizingly close to the energy scale of apparent gauge coupling unification in the Minimal Supersymmetric Standard Model, 2×10^{16} GeV. (See, Fig. 1.)

This way, the neutrino oscillation appears to provide us a unique window to physics at very high energies as their ‘‘leading order’’ effects. Indeed, theoretical estimates based on the seesaw mechanism in the grand unified theories [3] are practically confirmed!

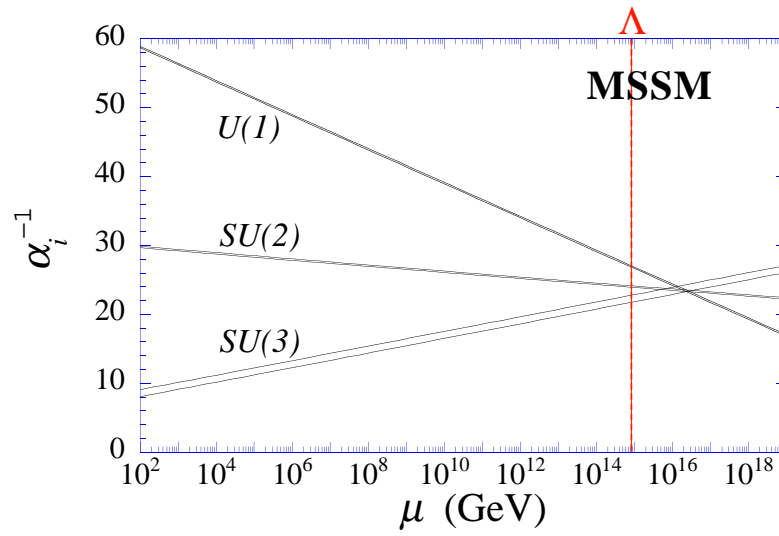


FIGURE 1. Apparent unification of gauge coupling unification in the MSSM at 2×10^{16} GeV, compared to the suggested scale of new physics from the neutrino oscillation data.

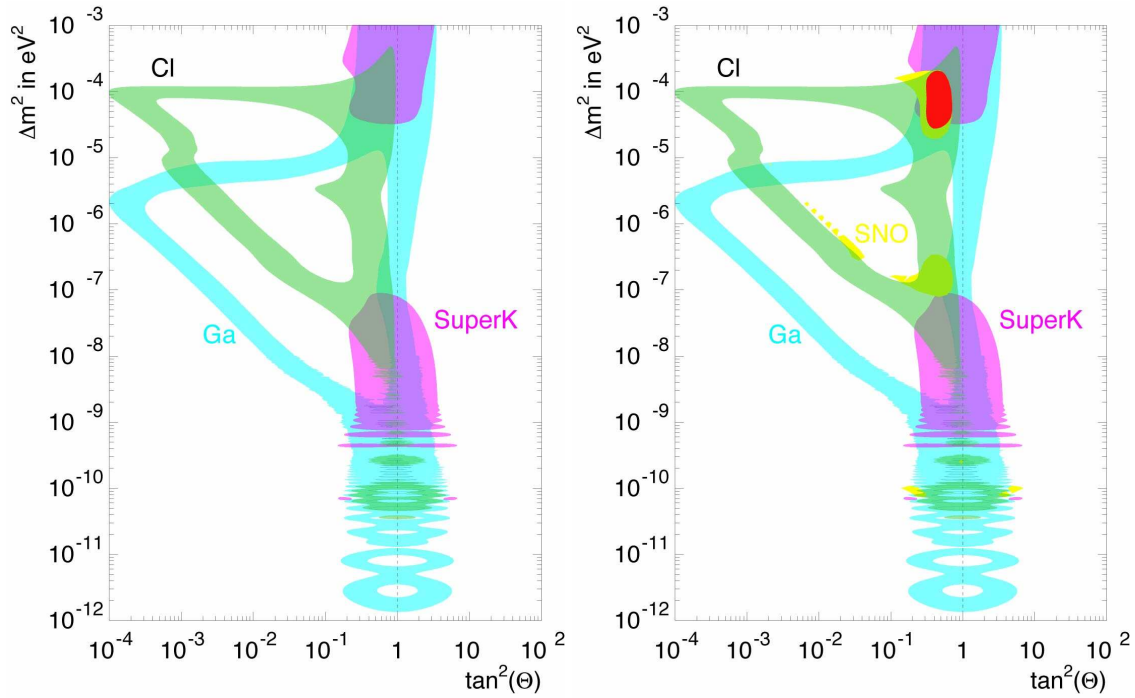


FIGURE 2. The progress in solar neutrino in the year 2002. Before March and after April [4].

PRESENT

The last year was an amazing year in neutrino physics. Before March, the situation of the solar neutrino data looked like the first plot in Fig. 2, and there had been overlaps

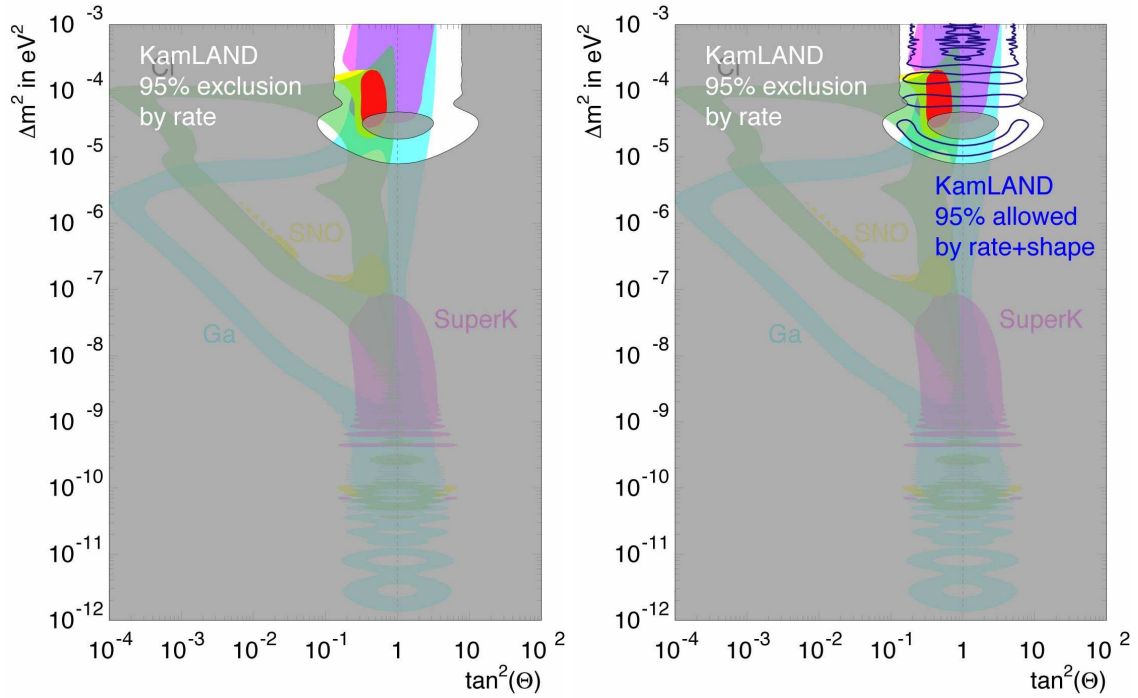


FIGURE 3. The comparison of the solar neutrino data and the reactor anti-neutrino data after December [4].

between SuperK, Homestake, and Gallium experiments in the LMA and LOW regions, some down in quasi-vacuum. After SNO neutral current result in April, the parameter space focused only on the LMA region shown in red in the second plot in Fig. 2. In December, KamLAND has excluded most of the parameter space as shown in the first plot in Fig. 3, while its preferred region (inside the blue contours in the second plot in Fig. 3) has consistent overlaps with the that preferred by the solar neutrino data. It was a tremendous convergence from the parameter space over many decades down to factors of a few.

It is useful to recall what a typical theorist used to say back around 1990.

- The solution to the solar neutrino problem must be the small mixing angle MSW solution because it is so beautiful.
- The natural scale for $\nu_\mu \rightarrow \nu_\tau$ oscillation is $\Delta m^2 \sim \text{eV}^2$ because it is the cosmologically interesting range.
- The angle θ_{23} must be of the same order of magnitude as V_{cb} because of the grand unification.
- The atmospheric neutrino anomaly must go away because it would require a large mixing angle to explain.

Needless to say, theorists have a very good track record in neutrino physics.

Indeed, the recent results from neutrino oscillation physics had surprised almost everybody. The prejudice has been that the mixing angles must be small because quark

TABLE 1. Prediction of different flavor symmetries on the neutrino mass-squared ratio and various mixing angles, taken from [6].

Model	parameters	d_{23}	$\Delta m_{12}^2/ \Delta m_{23}^2 $	U_{e3}	$\tan^2 \theta_{12}$	$\tan^2 \theta_{23}$
A	$\varepsilon = 1$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$
SA	$\varepsilon = \lambda$	$O(1)$	$O(d_{23}^2)$	$O(\lambda)$	$O(\lambda^2/d_{23}^2)$	$O(1)$
H _{II}	$\varepsilon = \lambda^2$	$O(\lambda^2)$	$O(\lambda^4)$	$O(\lambda^2)$	$O(1)$	$O(1)$
H _I	$\varepsilon = \lambda^2$	0	$O(\lambda^6)$	$O(\lambda^2)$	$O(1)$	$O(1)$
IH (LA)	$\varepsilon = \eta = \lambda$	$O(\lambda^4)$	$O(\lambda^2)$	$O(\lambda^2)$	$1+O(\lambda^2)$	$O(1)$
IH (LOW)	$\varepsilon = \eta = \lambda^2$	$O(\lambda^8)$	$O(\lambda^4)$	$O(\lambda^4)$	$1+O(\lambda^4)$	$O(1)$

mixing angles are small, and the masses must be hierarchical because both quarks and charged lepton masses are hierarchical. Given that the LMA is now chosen, all mixing angles are large except for U_{e3} that must be small-ish (but the current limit is not very strong, $|U_{e3}| \lesssim 0.2$).

The natural question then is if this newly discovered surprising pattern of neutrino masses and mixings require a new symmetry or any special structure to explain.

In fact, the big question has always been *what distinguishes flavor?* Three generations share exactly the same quantum numbers. Yet, they have such different masses. The hierarchy with small mixings means that there is a need for some kind of ordered structure. The “common sense” in quantum mechanics is that states with the same quantum numbers should have similar energy levels (*i.e.* masses) and mix significantly under small perturbations. The observed patterns go against this “common sense.” The hierarchical masses and small mixings among quarks and charged leptons had been a puzzle.

Therefore, there has been a strong suspicion that there is a new set of quantum numbers, *flavor quantum numbers*, that distinguish three generations of quarks and leptons. As Noether told us, a new quantum number requires a new symmetry, *flavor symmetry*. This new symmetry must allow the top quark Yukawa coupling because it is of the natural size, $y_t \simeq 1.0$. On the other hand, all the other Yukawa couplings are practically zero (as opposed to $O(1)$), and the flavor symmetry must forbid them. After the symmetry is broken by a small parameter, all the other Yukawa couplings become allowed, but suppressed [5]. The hope is to identify the underlying symmetry based on the data, similarly to what was done by Heisenberg (isospin) or Gell-Mann–Okubo (flavor $SU(3)$).

Indeed, the neutrino data had been already effective in narrowing down the possibilities of flavor symmetries. In Table 1, many proposed flavor symmetries are shown together with their predictions on the mass-squared ratio $\Delta m_{12}^2/\Delta m_{23}^2$, U_{e3} , θ_{12} , and θ_{23} , taken from [6] (October 2002).¹ Since then, models H_{II}, H_I, and IH (LOW) had been excluded by KamLAND.

¹ d_{23} in the model SA refers to a degree of accidental cancellation in the 23 sector, that is used to enhance θ_{12} .

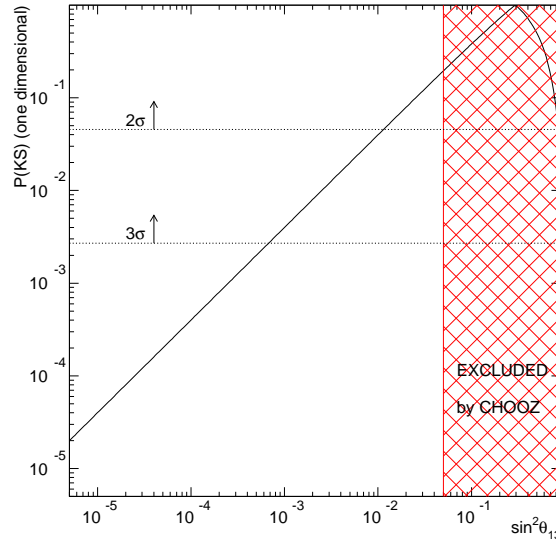


FIGURE 4. The one-dimensional KS probability on $\sin^2 \theta_{13}$ based on anarchy, *i.e.*, no fundamental distinction among three neutrinos. Taken from [9].

Among them, I liked the model A the best, because it is *mine* [7]. It is called *anarchy*, based on the idea that neutrinos are actually normal, while quarks and charged leptons aren't. As I mentioned already, the hierarchical masses and small mixing angles are against the “common sense,” while the neutrinos do not seem to have a large hierarchy and mix a lot. Maybe the *lack of flavor symmetry* can explain the data. Indeed, if there is no fundamental distinction among three neutrinos, or in other words if their flavor quantum numbers are all equal, the group theory of three-by-three unitary matrices uniquely determine the probability distribution of mixing angles [8]. Then all three angles, θ_{12} , θ_{23} , and θ_{13} are three random draws from the distribution $dP/dx \propto (1 - x)^{-1/2}$ for $x = \sin^2 2\theta$. Because it is peaked towards the maximal angle $x = 1$, it is very plausible that two draws come out large, while one of them comes down the tail (but not expected *way* down the tail). Indeed, the Kolgomorv–Smirnov test suggests that the probability that three random draws come out worse than the actual data is 64%, and hence the observed pattern is completely natural if there is *no* fundamental distinction among three generations [9]. On the other hand, θ_{13} is expected to be not too far below the current limit. The one-dimensional KS probability is $P(KS) = 4(\sin^2 \theta_{13} - \frac{1}{2} \sin^4 \theta_{13})$, and hence we expect $\sin^2 \theta_{13} > 0.013$ at “95% CL.” The size of the CP-violation $\sin \delta$ is distributed as $1/|\cos \theta|$, and hence is expected to be large.

FUTURE

Having discussed the impact of neutrino data on theory so far, it is clear what will be the critical measurements in the future.

- $\sin^2 2\theta_{23} = 1.00 \pm 0.01$? If it comes out that precisely maximal, it surely will require a new symmetry.
- $\sin^2 \theta_{13} < 0.01$? If so, electron-neutrino must have a different flavor quantum number from muon and tau neutrinos.
- Normal or inverted hierarchy? Most flavor symmetries predict the normal hierarchy, but theorists had been wrong!
- CP Violation? Even though the CP violation in neutrino oscillation may not prove the relevant CP violation for leptogenesis, it will at least make it very plausible.

After going through the critical measurements, we hope to determine the underlying flavor symmetries behind neutrinos (and flavor in general). Then comes an even bigger question: can we understand the dynamics behind the flavor symmetry? In the case of the strong interaction, isospin and flavor $SU(3)$ are the flavor symmetries, while the QCD is the dynamics. Can we get to the same level? This question will depend crucially on what we will find at the TeV-scale. If it is supersymmetry, the answer may be anomalous $U(1)$ gauge symmetry with the Green–Schwarz mechanism from the string theory [10]. If it is extra dimensions, the answer may be physical dislocation of different particles within a thick brane [11]. If it is technicolor, the answer may be new broken gauge symmetries at 100 TeV scale [12].

Of course one shouldn't forget LSND [13] because no theory fit the data very well. It is true that most theorists do not take the LSND evidence seriously at this moment, only data will decide. Currently all explanations have difficulties: sterile neutrino(s) [14], CPT violation [15], lepton-number violating muon decay [16]. But if any of them will turn out to be true, it will have a huge impact on theory.

CONCLUSION

Neutrino oscillation physics has had big impact on theory already. Yet, there is a lot more to learn. The (precise) measurements of θ_{23} , θ_{13} , the type of hierarchy, and the CP violation, will have critical impact. Through these measurements, we hope to determine the symmetries behind the neutrino masses and mixings or flavor in general. In conjunction with data from the energy frontier, we may even have access to understand dynamics behind the flavor. Depending on how things will turn out, there may well be even more surprises.

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